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OPTIMUM CITY SIZE: THE EXTERNAL DISECONOMY QUESTION*

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This paper discusses whether market achieved city size is greater or less than optimum city size. The divergence between optimum and achieved city size is due to external diseconomies such as pollution. Imposing an optimal tax on pollution may not, as is commonly thought, cause even an initial reduction in output of the polluting good. Moreover, the paper shows even if output initially falls with optimal taxation, the corresponding reduction in pollution and shift toward consumption of non-polluting goods will make city inhabitants better off. The increased welfare of city inhabitants will result in immigration to the city.

A popularly held idea is that cities, in particular big cities, are too large and that optimum size cities would be smaller.[1] Non-optimality arises because of unpriced production or consumption externalities such as noise, air, or water pollution and congestion, which if properly priced would lead to a reduction in city sizes. This paper disputes this conclusion for numerous reasons. For illustrative purposes, the paper concentrates on an analysis of air and water pollution.

The first task is to formulate the problem in a simple but appropriate context. Firms produce an export good in a city. Due to a production input such as sulphur bearing coal or petroleum, the production of the export good results in air or water borne emissions such as sulphur dioxide and phenols.[2] This pollution may affect production efficiency of firms (or households); or, as will be the case in our principal example, it may just enter utility functions of people living in the city. Concerning this, one might hypothesize that since many work places are air conditioned and purified primarily to stabilize seasonal fluctuations in temperature, air pollution in general does not enter production functions or affect efficiency significantly. Similarly, water pollution little affects the efficiency of water use in industrial processes. However pollution certainly affects consumption activity either directly entering the utility function or affecting efficiency of household production. That is, it is part of the amenity income from living in a city.

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1. See Sandquist (1970) or Mills and de Ferranti (1970). Mills and de Ferranti also question the standard conclusion.

2. See Kneese (1971) for a description of the multiple forms and effects of pollution.

What happens if these externalities are priced to equate social and private marginal costs (assuming they can be priced[3])? The usual analysis states that, if production of a good results in external diseconomies, optimally taxing this good will result in a decline in its production and an increase in production of other goods. The inference is then made that if production of the polluting good falls, the sizes of cities producing that good fall. In contrast, our analysis leads to the following conclusions:

(1) Given the recent work of Plott (1966), Tolley (1969), and Schall (1971) on externalities, in Part I the paper will show that the standard result of reduced production of a polluting good when optimally taxed is just one of several results arising from a comprehensive analysis of the problem. Production of the good may well rise. If production of the good rises then, given the nature of the conditions under which production increases and the analysis in Part II of the paper, it seems likely city sizes will increase following the imposition of the tax.

(2) Even if economy production of the good falls, there is no reason to assume sizes of cities producing the good will fall. Two alternatives are possible.

(a) City production of the good will fall but city size will stay the same or increase. The conclusion results from a comprehensive analysis of not only the effect of the tax upon the production of the polluting good but its effect upon the production of other goods and the effect of reduced pollution upon the welfare of the city inhabitants. We conclude that, with the imposition of the tax and initial decline in production of the

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3. See Baumol and Oates (1971) and Bohm (1971) for a description of pricing schemes and other methods of pollution control.

polluting good, the welfare of city inhabitants in cities producing the polluting good rises relative to the rest of the economy and the cities experience an influx of people. With this influx of people it is conceivable that production of the polluting good in <u>some</u> cities will rise beyond the pre-tax levels.

(b) If production of the good rises in some cities, how does this mesh with the fact that economy production falls under the standard analysis? First, economy production will fall under the standard analysis because the tax makes the good more expensive to produce. Second, inhabitants in cities producing the good will be made better off with the imposition of the tax causing an influx of people into those cities and perhaps causing output of the polluting good to rise in some cities. If city production rises in some cities, other cities will cease to produce the polluting good to effect reduced economy production of the good. The idea is simply that the optimal size production-consumption unit or city producing the polluting good has risen but economy output has fallen. With increased optimal city size, the number of cities producing the good falls.

The City

Before proceeding to the main analysis, the concept of a city and of a system of cities in an economy is examined. In discussing optimum city size it is very important to understand what a city is and does and how it relates to the total economy.

Cities produce a bundle of goods. As characterized by Mills (1967), in even the most simple model, cities produce an export good or bundle of goods, housing, and a commuting good. Agglomeration in cities occurs because of scale economies in producing the export good. Workers work in the city center producing the export good and commute from their home site to the city center and back each day. As city size increases, per person commuting costs rise due to increasing spatial distances that workers must commute and rising congestion on the roads. As expressed in Mills (1967) and in Henderson (1972), these rising commuting costs with agglomeration may offset the benefits of agglomeration or scale economy exploitation in traded good production. That is, there is an efficient city size for production and consumption.

Extending this analysis, Henderson (1972) then shows if there is an efficient city size where net benefits of agglomeration are maximized,[4] there may be multiple cities producing the same bundle of goods.[5] Further, as has been asserted by numerous people (e.g., Lösch (1954) and Beckman (1968)), Henderson also shows there are different types of cities in the economy. Each type of city specializes in the production of a different good or bundle of export goods.[6]

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4. Henderson does not state the market achieves this efficient city size. His analysis and any reasons for non-optimality are independent of the analysis in this paper or external diseconomies.

5. This result can be derived in a Loschian (1954) world where efficient city size is limited by the increasing transport costs of supplying larger and larger rural areas as a city size increases.

6. Henderson's (1972) reasoning is as follows. If the production of two export goods or bundles of export goods involve no benefits from a common location such as utilizing a common specialized labour force or locationally fixed intermediate input, then it may be beneficial to locate the production of these goods in different cities. Locating their production together increases the labour force that must be housed and that adds to average commuting costs in the city while not increasing efficiency in the production of the traded goods. Separating the production of the two goods into different cities lowers the increase in per person commuting costs <u>relative</u> to scale economy exploitation <u>per unit of traded good</u> output, as <u>traded</u> good production and city size rise. Separating the two industries of course involves further transportation expenditures due to inter-city trade and a complete analysis would have to account for these costs.

These propositions indicate there are a system of cities in an economy with different types of cities producing different bundles of export goods. Within each type of city, there may be one or multiple cities depending on economy or world demand for the export goods of that city type, their costs of transportation in trade (the Loschian (1954) limitation on city size), and their production technology, in particular, the degree of scale economies (the extent of city agglomeration economies).

This above discussion relates to the externality question and our opening remarks in the following way. Suppose there are two types of cities in the economy specializing in a manufactured export good and a service export good. Factors are mobile between all cities. The production of the manufactured good involves external diseconomies or pollution and optimal taxation of the externalities may reduce output of the good in the economy increasing output of the service good. Although manufacturing production falls, the efficiency of manufacturing cities as production-consumption units producing housing, household services, the manufactured good, and amenity income may increase with the reduction of the externality in those cities. If this is the case and given initial multiple manufacturing cities, average city size and even production of manufactured goods in each city may increase. Then lower economy output of that traded good occurs with a reduction in the number of manufacturing cities.[7]

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7. If the city-specialization argument is not accepted and it is viewed that cities, under our simple assumptions, would produce all goods, our argument is even stronger! Optimally pricing the good causing externalities will only change the production ratios of goods in the economy and city. This says nothing, *a priori*, about the effect on city size.

Part I: Externalities in Production

In this section we formally specify the nature of pollution and then examine what happens if pollution is optimally priced. In particular we examine for fixed factor endowments in a country or in a city whether if pollution is priced the output of the polluting good will fall and also whether labour employment in that industry will fall. For city size held constant, we indicate that, by optimally pricing the good, output of all goods in the city may increase with pollution reduced. Since the welfare and income of city inhabitants must then be improved relative to the rest of the country city size should increase through immigration. Also country as well as city output of the good may rise. Until Part II we do not fully examine the change in equilibrium in a city, but our comments will be sufficient to make our point.

The formal analysis is carried in terms of a representative city but implications for the country as a whole are followed through. Our representative city produces an export good, X, and a non-traded good such as housing, Y. Firms in the X industry utilize an input that results in pollution which, in turn, affects the efficiency of production in the X and Y industry. Specifically, a firm's production function for the city's export good is:

$$x_i = g_i(P)f_i(L_i, N_i) \qquad i = 1, \ldots, n \tag{1}$$

$$\partial f_i/\partial N_i > 0; \quad \partial f_i/\partial L_i > 0; \quad (dg_i/dP)(\partial P/\partial N_j) < 0, \quad i,j = 1, \ldots, n$$

where x_i is the ith firm's output, L_i and N_i are inputs of labour and an imported resource, and P is pollution. f_i is the constant returns to scale production function of the firm. $g_i(P)$ is a production function shifter that affects firms' efficiency in a Hick's neutral fashion with respect to factor inputs; e.g., pollution corrodes machinery as well as irritating workers. Our specification is essentially one of industry scale dis-economies external to the firm. Conforming to Chipman's (1970) excellent article we assume that firms do not account for the effect of their own production upon pollution and their own efficiency. All pollution including the firm's own pollution is an unpriced externality to the firm. Assuming perfect competition and given that the firms perceive a constant returns to scale production function, factor payments based upon perceived marginal productivity exhaust firm revenue. (The fact that to have urban agglomeration X would normally be produced with industry (or firm) scale economies is ignored at the margin of current output; the analysis should not be qualitatively distorted by this. Note that scale economies are needed to explain why firms remain in the city crowded together despite pollution, which they could avoid by spatially separating themselves.)

The industry production function for Y is

$$Y = G(P)F(L_Y) \tag{2}$$

$$\partial F/\partial L_Y > 0; \qquad dG/dP \cdot \partial P/\partial N_i < 0, \qquad i = 1, \ldots, n$$

should be $dG/dP < 0$ shouldn't it.

$G(P)$, caused by city production of X, is also a production function shifter. Although Y is called housing it could be expanded to include many other goods such as household services.

(Our production specification is chosen for analytic convenience. In the alternative unpriced input, specification, in addition to the externality distortion, we have an unpriced input distortion since P would be an unpriced input in the f_i and F production functions in equations (1) and (2). This would mean that P might earn a negative rent which, in perfect competition in output and factor markets, would have to be "assigned" to the earnings of one or both of the other factors creating further distortions such as analyzed by Worcester (1969) and Meade (1952).[8] There has been some debate on how to deal with an unpriced factor[9] and we avoid this debate by using the production function specification where the only distortion is the externality itself. We do footnote interesting results that would arise if we treated the problem as an unpriced input.)

Pollution such as sulphur dioxide or phenol emissions is created due to the use of a resource such as coal or petroleum by firms in the X industry and is a public good. That is

$$P = P(N_1, N_2 \ldots N_n) \qquad (3)$$

?

where

$$\partial P/\partial N_i \gtreqless \partial P/\partial N_j , \qquad i \neq j$$

All firms employing N contribute to a stock of pollution that affects each firm's production as an external diseconomy. (This stock formulation is from Tolley (1969).) Note that firms' marginal contributions to pollution, $\partial P/\partial N_i$, may differ depending on relative spatial locations of firms and prevailing winds or on individual firm production functions. The production function of P is not necessarily homogeneous of degree one. At low levels

two points ① Not linear homog. ② Cont. units $P = P(\bar{N})$ $\bar{N} = \Sigma N_i$

8. It is possible with an unpriced factor which is non-rivalrous that the rent will be zero.

9. Meade (1952) "assigns" the rent of the unpriced factor to capital which, if the unpriced factor is an external diseconomy, is a negative rent. Given perfect capital markets, the negative rent means capital is underutilized, since its social marginal product is less than the market price which is equal to the private marginal product plus the negative rent. To equate private and social marginal products, Meade suggest subsidizing capital, from the tax proceeds received from taxing the source of the externality. Mishan (1965) has criticized both the automatic assignment of rent to capital versus other factors, as well as subsidizing capital rather than firm output.

of X output, additions of N add little to pollution. At higher levels the natural absorption of pollution by the atmosphere or water from chemical breakdown or spatial distribution by wind or water currents is slowed and additions of N contribute significantly to pollution.

As Plott (1966) points out, that N_i, not x_i, is the cause of pollution is crucial. N_i, not x_i, is the taxable externality source. Even, if x_i production falls with N_i taxation, this does not mean labour employment will fall, given factor substitution.

We are now ready to analyze equilibrium versus optimum factor pricing, production conditions and output. It is assumed readers are familiar with the details of the two (equilibrium and social optimum) constrained optimization problems and we turn directly to the results. In competitive equilibrium, factors are paid the value of their perceived marginal products.

is this a value

Following Chipman (1970), from equation (1) for x_i, where we just consider two firms for expositional simplicity,

$$p_L = p_X g_i(P)\partial f_i/\partial L_i \qquad i = 1,2 \tag{4}$$

$$p_N = p_X g_i(P)\partial f_i/\partial N_i \qquad i = 1,2 \tag{5}$$

where all p's are prices. For Y, from equation (2)

$$p_L = p_Y G(P)dF/dL_Y \tag{6}$$

Equations (4) and (6) also describe the value of social marginal products for labour. However, the value of social marginal products for N are different from equation (5). N_i affects production efficiency of firms in the X and Y industries by contributing to pollution which in turn acts as a production function shifter in these industries. For firms 1 and 2 the

the social marginal products of N are

$$\text{firm 1:} \quad \hat{p}_N = p_X g_1(P)\partial f_1/\partial N_1$$
$$+ [p_X(dg_1/dP\cdot\partial P/\partial N_1\cdot f_1 + dg_2/dP\cdot\partial P/\partial N_1\cdot f_2) + p_Y dG/dP\cdot\partial P/\partial N_1\cdot F] \qquad (7)$$

$$\text{firm 2:} \quad p_N = p_X g_2(P)\partial f_2/\partial N_2$$
$$+ [p_X(dg_2/dP\cdot\partial P/\partial N_2\cdot f_2 + dg_1/dP\cdot\partial P/\partial N_2\cdot f_1) + p_Y dG/dP\cdot\partial P/\partial N_2\cdot F] \qquad (8)$$

where f_1, f_2 and F are abbreviations for $f_i(L_i, N_i)$ and $F(L_Y)$. The first and second terms in the square brackets indicate the negative effect in the form of a production function shift of employing additional N_i in the X industry and the third term represents the negative effect in the Y industry. In each case, per unit use of N should be taxed by the amount in the square brackets to equate both factor price with social marginal product and social and private marginal costs.[10] The standard result following the imposition of the tax is that, with the increase in p_N, the use of N and output of X falls. There are three exceptions to this.

Pollution in a city may be so extensive that additional use of N may actually reduce total output. Although $\partial f_i/\partial N_i > 0$, additions [reductions] to N may add [remove] so much to pollution and hence so affect production efficiency through large marginal reductions in the g_i and G terms in equations (1) and (2), that, in net, total output will fall [rise]. In other words in competitive equilibrium, equations (7) and (8) for the social marginal product of N may be negative. If N is optimally priced or taxed the importation and use of N will fall in the city. For initial reductions in

10. See Meade (1952) in his atmosphere case or Chipman (1970).

N total productivity or output will rise because the true marginal product of N is negative. In the social optimum, total productivity or output holding the city labour force fixed may be higher or lower depending on how negative the social marginal product of N originally was, what its optimum level is, and how the proportions of X and Y in a consumer index of total output change given the increase in input cost in X production.[11] (Compare this with the textbook curves for marginal and total product as N changes for a fixed L in producing one good--if the marginal product of N is negative, reducing its use will increase total product.)

In final equilibrium both X and Y output could rise for the same labour force and reduced use of N or output of X could fall and of Y could rise given demand conditions.

Whether X and Y both rise or only Y rises depends also on whether the social marginal product of N in X production alone is negative--on whether the first term in equations (7) and (8) is outweighed by the first two terms alone in the square brackets. In that case X as well as Y production may rise in the social optimum. In either case since the index of total output for the same city labour force could rise with reduced use of N and pollution, city inhabitants may be unambiguously better off. If the city inhabitants initially had equalized welfare relative to people in other parts of the country the increase in welfare would lead to immigration and an increase in city size. This type of argument will be extended in part II more rigorously. The point is city size and city and country production of X all may rise since

11. Tolley (1969) has a similar example but does not make the negative social marginal product condition explicit. To quote Tolley, "A manifestation of the gain from reducing the externality is likely to be that the production function shift....lowering costs, is greater than the effect of the tax as a cost-raising item."

the initial point of production is inefficient[12] as well as non-optimal.

Our second situation where optimally taxing a polluting good may lead to a rise in its output also involves an inefficient initial equilibrium. A simple example will illustrate the situation. There are two firms initially producing equal amounts of X and both experience equal amounts of pollution. However only one firm creates pollution due to differences in production functions or spatial locations--for example the "non"-polluting firm may be downwind on the edge of the city. If all production were shifted to the non-polluting firm, for the same inputs, total output of X (and Y) would rise.

In other words, firms may not contribute equally to pollution at the margin given either their production technology or relative spatial locations; that is $\partial P/\partial N_1 \neq \partial P/\partial N_2$. Alternatively, $\partial g_1/\partial P \neq \partial g_2/\partial P$; or pollution affects efficiency of firms differently for the same reasons. If either is the case equations (7) and (8) for social marginal product will not be equal in competitive equilibrium. (Of course, private marginal products will be equal.) Suppose the social marginal product of N in firm 2 is greater than in firm 1. If we held employment in Y and city labour force fixed, and moved some of N_1 to firm 2, output of X would rise for the same inputs.

All this implies that we are not on the Pareto-efficient transformation surface in competitive equilibrium where equations (5) but not (7) and (8) are equalized. If we impose a Pareto-optimal tax on the use of N we

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12. By inefficient, we mean if resources were diverted from national production of N, not only could production of X and Y or, at least, an index of total output of the two goods rise, but the diverted resources could be used in production of other additional consumer goods. For the same initial resources by rearranging production conditions, total output of all goods could rise.

move from the private competitive equilibrium interior to the transformation curve in figure 1 to the social optimum on the transformation curve for the city. Initially we are at the production point, S. By taxing N_i in the production of x_1 and x_2, we (a) rearrange production in the X industry efficiently, shifting us to the transformation curve and (b) eliminate the divergence between private and social marginal costs caused by the X industry polluting itself and the Y industry. Usually only (b) is considered which would indicate a fall in X production. Including (a) may cause both outputs to rise, as at production point T.[13] Rearranging N usage and X production between firms to those polluting less or being less affected by pollution, X output may rise despite the tax on N and its fall in total usage. Note

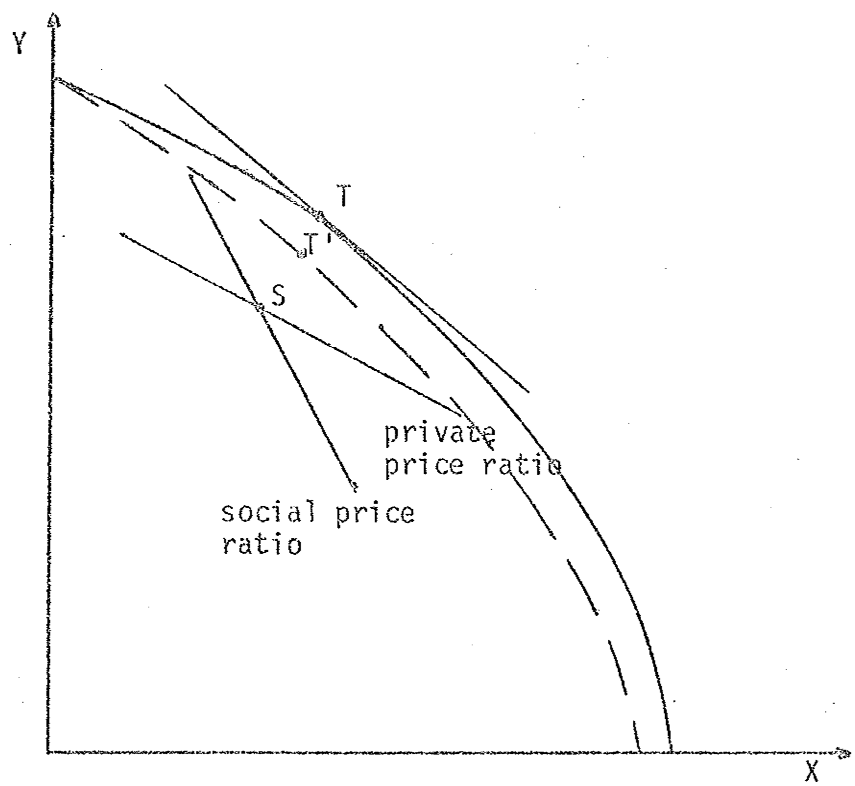

Figure 1. Optimal Production

13. This argument utilizes Schall's (1972) work.

that the decline in N usage by the city will actually shift the transformation surface back; equilibrium will be at, say T'. However the total output and welfare of city inhabitants may still rise, resulting in immigration and an increase in city size. These arguments will be reinforced following the analysis in Part II of the paper.

Our third situation is a more economically technical one and our discussion is brief. As Plott (1966) points out because pollution is caused by the use of an input, taxing the input N may still lead neither to a fall in X output nor in use of labour. The first case as shown in Bear (1965) occurs if N is an "inferior" input. Taxing and reducing the use of N and raising its price, can lead to an increase in X output for p_X and p_L held constant. Secondly, raising the price of N in a partial equilibrium model (our city or type of city is a small portion of the economy) may or may not reduce the use of labour in the X industry after factor substitution if (a) N and L are "complements" (an increase in the quantity of one factor does not increase the marginal product of the other factor) for p_X held constant[14] or (b) given factors are substitutes, the size of the elasticity of substitution is greater than the elasticity of the demand for the product, for p_X allowed to vary (see Allen (1968) pp. 371-374).[15] Admittedly these conditions are not expected to normally hold; but, if they do, economy labour usage in

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14. See, for example, Henderson and Quandt (1971), p. 70.

15. Allen's (1968) condition occurs when, for $M = \frac{P_N N}{P_X X}$

σ = the elasticity of substitution, and η = elasticity of demand,

$$(\partial L/\partial p_N)\cdot(p_N/L) = m(\sigma-\eta) > 0, \text{ if } \sigma > \eta.$$

X and city size may well rise.[16]

In this section we have shown that optimally taxing the use of the factor causing an external diseconomy does not necessarily lead to a decline in the output of the industry. Of particular interest is the very realistic situation raised by Schall (1971) where firms pollute or are affected by pollution in differing degrees. Taxing the factor source of pollution while perhaps reducing use of that factor may lead to a more efficient allocation of output between firms and a rise in total output.

Part II: Efficient City Size

We now turn to a comprehensive examination of the optimum city or production-consumption unit size. We deal with the standard situation where taxing output leads to a decrease in output of the industry. Introducing any of the cases in Part I would only strengthen our arguments, as will become obvious. For simplicity we assume externalities caused by production only affect consumption or production of leisure activities. The effect is summarized by placing pollution directly in the utility function.

The city to be considered is a representative city of the type that produces and exports good X to national and/or international markets. There may be multiple cities producing X and other types of cities exporting other goods. In addition to the export good, a variety of non-traded goods and

16. If the problem is formulated as an unpriced input, the analysis is even more complicated. For example, if there is a negative rent from the unpriced pollution, and if it is "assigned" to labour in the X and Y industry leading to too high a social marginal product of labour and too low labour usage, following Meade (1952), we would subsidize labour usage (or firm output) from the tax proceeds from N taxation. Then the factor substitution effect is strongly in favour of raising total labour usage in X production!

services are produced in the city, summarized in a bundle of goods Y. In discussing the effect of optimally taxing pollution in this section we develop a framework to evaluate in net the effect of loss in output of the polluting good, benefits of pollution reduction, pollution tax proceeds, etc.

Factor inputs are labour and a resource input, N. N is purchased at a fixed price to the city from other areas such as processing towns or mines. Labour is perfectly mobile in the economy and moves to equalize utility levels. By using utility levels rather than wage rates we capture the effect of not only wage differentials between cities but also differentials in prices of non-traded goods and amenity income or, specifically in our example, pollution.

Currently the city is in production and consumption equilibrium. Its labourers have equalized utility levels with labourers in other cities and its trade is balanced. However the use of N in producing X causes pollution which enters people's utility functions as a public good (or bad). To move towards a Pareto-optimum, the use of N is taxed and the proceeds redistributed to the city's inhabitants. This tax is introduced holding city population initially fixed. As will be shown, the tax leads to a fall in N usage and X output; it also leads to a fall in pollution. We will examine whether in the new initial equilibrium utility levels have risen or fallen in the city and hence whether labour will migrate to or from the city, changing city size and adjusting utility levels to those prevailing in national labour markets. If there is immigration, in the final equilibrium city output of X may increase relative to the initial levels.

Assuming X production in the economy falls, if our representative city and its production of X increases in size then some other cities producing the good will either decline in size or more likely disappear. Which services are produced in the city, summarized in a bundle of goods Y. In discussing the effect of optimally taxing pollution in this section we develop a framework to evaluate in net the effect of loss in output of the polluting good, benefits of pollution reduction, pollution tax proceeds, etc.

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Assuming X production in the economy falls, if our representative city and its production of X increases in size then some other cities producing the good will either decline in size or more likely disappear. Which cities survive and increase in size and which disappear, given our information, is indeterminant. A more sophisticated model would incorporate the analysis of the previous section. For example, those cities with firms that are the worst polluters due to inefficiency or regional atmospheric stagnation, would be the ones most adversely affected and would tend to disappear.

The production functions for the X and Y industry are respectively

$$X = X(L_X, N) \tag{9}$$

$$Y = Y(L_Y) \tag{10}$$

These functions are homogeneous of degree one. (Although X and Y as previously mentioned are probably produced with non-constant returns to scale, this effect is ignored in the algebraic presentation, although it is commented on.)

Labour is fully employed in the city or

$$L_X + L_Y = L \tag{11}$$

The use of N in the X industry results in pollution which enters the utility function as an unpriced public good. It is assumed all firms pollute equally and that pollution results from the use of N or

$$P = g(N) \tag{12}$$

City population is equated to labour force for simplicity. All consumers are assumed to have identical tastes and income. Per person utility is defined by

$$U_i = U_i\left(\frac{X^C}{L}, \frac{Y}{L}, \frac{Z^C}{L}, P\right) \qquad i = 1 \ldots 2 \tag{13}$$

Superscripts C refer to goods consumed in the city when the distinction between consumed and produced applies. The city exports X and imports Z and the input N. The balance of trade equation for the city is

$$p_X(X-X^C) = p_Z Z^C + p_N N \tag{14}$$

In other words, city income or expenditures equal

$$I = p_L L = p_X X + p_Y Y - p_N N \tag{15}$$

At present pollution is an unpriced public good entering people's utility functions. By pricing this externality we will achieve a potential Pareto optimum relative to current equilibrium. To price the use of N, we tax it redistributing the proceeds to the inhabitants.

To discover the effects of imposing the tax, we employ a general equilibrium model of the Jones (1965) type. We assume p_X, p_N (excluding the tax), and L are all initially fixed. Although we will only explicitly use the directional nature (positive) of the tax on N, for completeness we calculate the tax here. We do not explicitly distinguish between the initial tax imposed at the old equilibrium and the equilibrium (and presumably smaller) tax in the Pareto-optimum solution.

After the imposition of the tax which is redistributed to labourers, consumers maximize utility subject to a budget constraint.

$$I/L = p_X\left(\frac{X^C}{L}\right) + p_Y\left(\frac{Y}{L}\right) + p_Z\left(\frac{Z^C}{L}\right) + \left(\frac{p_P}{L}\right)P, \quad p_P < 0 \tag{16}$$

where p_P is the price per unit of pollution and p_P/L, the per person negative head tax. From the constrained maximization problem, the Samuelsonian public good condition says

$$p_P/p_X = \frac{U_P L}{U_X}, \quad \partial U/\partial X = U_X, \quad \frac{\partial U}{\partial P} = U_P \tag{17}$$

or the price ratio equals the sum of the marginal utilities of the public good normalized by the marginal utility of a private good. If P and N were the original quantities of pollution and N, then

$$p_P(P-\Delta P) = -t(N-\Delta N), \qquad t > 0 \tag{18}$$

which says that proceeds from the post-tax employment of N equal subsidies (or negative head tax) paid to labourers, where Δ's refer to changes. Rearranging and using equations (12), (17), and (18), we get the tax,

$$t = -(1-\overset{*}{N})^{-1}(P/N-g_N\overset{*}{N})(p_X \frac{U_P}{U_X} L) > 0 \tag{19}$$

where asterisk refers to rates of change and $g_N = dg(N)/dN = dP/dN$.

To determine the effect of the tax, t, upon equilibrium we employ Jones' (1965) equations of change. Calculus or small changes in variables are used with all the attendant qualifications on the nature of such solutions --however in the paper we are only concerned with directional indications not quantitative estimates. There is no reason to expect large rather than marginal changes would involve a reversal in the directional changes predicted in this paper. Those unfamiliar with Jones' model will find the accompanying discussion revealing. Our basic equations are as follows:

$$\begin{aligned} a_{XL}X + a_{YL}Y &= L \\ a_{XN}X &= N \end{aligned} \tag{20}$$

where a_{ij} refer to the amount of input j into a unit of i output. Equations (20) say all resources are employed.

$$\begin{aligned} a_{XL}p_L + a_{XN}p_N &= p_X \\ a_{XL}p_L &= p_Y \end{aligned} \tag{21}$$

which say factor payments to inputs used to produce a unit of output equal revenue from that output or equal price. There is perfect competition. Following Jones, the equations of change for (20) and (21) are

$$\begin{bmatrix} \lambda_{XL} & \lambda_{YL} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \overset{*}{X} \\ \overset{*}{Y} \end{bmatrix} = \begin{bmatrix} 0 \\ \overset{*}{N} \end{bmatrix} - \begin{bmatrix} \lambda_{XL} & \overset{*}{a}_{XL} \\ \overset{*}{a}_{XN} & \end{bmatrix} \tag{22}$$

$$\begin{bmatrix} \theta_{XL} & \theta_{XN} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \overset{*}{p}_L \\ \overset{*}{p}_N \end{bmatrix} = \begin{bmatrix} 0 \\ \overset{*}{p}_Y \end{bmatrix} + \begin{bmatrix} \theta_{XL}\overset{*}{a}_{XL} + \theta_{XN}\overset{*}{a}_{XN} \\ 0 \end{bmatrix} \tag{23}$$

where an asterisk refers to a rate of change, where λ_{ij} refers to the fraction of the jth factor used in the production of the ith good and θ_{ij} refers to the jth factor's share in the output revenue of the ith good. Note that $\overset{*}{a}_{YL} = 0$ by assumption of constant returns to scale and one input in Y. Note that $\lambda_{XN} = 1$ and $\lambda_{YN} = 0$ since all N is used in X production; also $\theta_{YL} = 1$ and $\theta_{YN} = 0$ since only labour is used in Y production. Note finally that by assumption $\overset{*}{L} = 0$ initially, $\overset{*}{p}_N = \frac{t}{p_N}$ or the tax divided by the city import price of N, and $\overset{*}{p}_X = 0$ initially or p_X is determined in national and international markets.

With respect to equation (23), Jones reasons by principles of cost minimization that

$$\theta_{XL}\overset{*}{a}_{XL} + \theta_{XN}\overset{*}{a}_{XN} = 0 \tag{24}$$

Solving equation (23) yields

$$\overset{*}{p}_L = p_Y^* = -\frac{\theta_{XN}}{\theta_{XL}}\, p_N^* < 0 \tag{25}$$

Since p_X is fixed, the rise in p_N indicates p_L must fall to maintain p_X; and $\overset{*}{p}_L$ in turn causes p_Y to decline. In other words the rise in the price of N causes substitution away from its use, a decline in N/L, and hence a decline in p_L.

We define an elasticity of substitution $\sigma_X = \dfrac{a_{XL}^* - a_{XN}^*}{p_N^* - p_L^*}$. Using σ_X, equation (24), and substituting in equation (25) for p_N^* and p_L^*, equation (22) becomes

$$\begin{bmatrix} \lambda_{XL} & \lambda_{YL} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \overset{*}{X} \\ \overset{*}{Y} \end{bmatrix} = \begin{bmatrix} 0 \\ \overset{*}{N} \end{bmatrix} + \sigma_X p_N^* \begin{bmatrix} -\lambda_{XL} \; \theta_{XN}/\theta_{XL} \\ 1 \end{bmatrix} \tag{26}$$

To complete our model we need to introduce a demand side. We specify demand equations for Z and Y (the one for X is superfluous).

$$Y = Y(I, p_Y, p_X, p_Z), \qquad Z^C = Z(I, p_Y, p_X, p_Z)$$

Taking changes we get

$$\overset{*}{Y} = \eta_{Y,I}\,\overset{*}{I} + \eta_{Y,p_Y}\,\overset{*}{p}_Y \quad , \quad \overset{*}{p}_X = \overset{*}{p}_Z = 0 \tag{27}$$

$$\overset{*}{Z}{}^C = \eta_{Z,I}\,\overset{*}{I} + \eta_{Z,p_Y}\,\overset{*}{p}_Y \tag{28}$$

where $\eta_{Y,I}$ and η_{Y,p_Y} (and η_{Z,p_Y}) are respectively income and price (money income held constant) elasticities of demand and $\overset{*}{p}_Y$ is given in (25). $\overset{*}{I}$ can be derived from equation (15) or

$$\overset{*}{I} = \frac{p_X X}{I}\,\overset{*}{X} + \lambda_{YL}\,\overset{*}{Y} + \lambda_{YL}\overset{*}{p}_Y - \frac{p_N N}{I}\,\overset{*}{N} \tag{29}$$

Note that the pollution taxes are returned to city inhabitants as pollution compensation and therefore p_N, the price paid for imports of N is unchanged. We define k_Y, the relative amount of the initial consumer budget spent on Y where $k_Y = \dfrac{p_Y Y}{p_L L} = \dfrac{p_L L_Y}{p_L L} = L_Y/L$. Combining (27), (29), and (25) we get

$$\overset{*}{Y} = (1-k_Y\eta_{Y,I}) = \frac{p_X X}{I}\overset{*}{X} - \frac{p_N N}{I}\eta_{Y,I}\overset{*}{N} + \eta^S_{Y,p_Y}\overset{*}{p}_Y \qquad (30)$$

where $\eta^S_{Y,p_Y} = \eta_{Y,p_Y} + k_Y\eta_{Y,I} < 0$, the Hicks-Slutsky substitution elasticity.

Since prices p_X and p_Z are fixed, to solve for equilibrium <u>outputs</u> (not consumption) we do not need demand equations for X and Z. Equations (26) and (30) determine outputs. Combining (26) and (30) yields

$$\begin{bmatrix} \lambda_{XL} & \lambda_{YL} & 0 \\ 1 & 0 & -1 \\ \left(-\frac{p_X X}{I}\eta_{Y,I}\right) & (1-k_Y\eta_{Y,I}) & \left(\eta_{Y,I}\frac{p_N N}{I}\right) \end{bmatrix} \begin{bmatrix} \overset{*}{X} \\ \overset{*}{Y} \\ \overset{*}{N} \end{bmatrix} = \overset{*}{p}_N \begin{bmatrix} -\lambda_{XL}\frac{\theta_{XN}}{\theta_{XL}}\sigma_X \\ \sigma_X \\ -\eta^S_{Y,p_Y}\frac{\theta_{XN}}{\theta_{XL}} \end{bmatrix} \qquad (31)$$

Solving, using Cramer's rule

$$\overset{*}{X} = \overset{*}{p}_N\left(\frac{\eta^S_{Y,p_Y}\frac{\theta_{XN}}{\theta_{XL}}k_Y - \lambda_{XL}\frac{\theta_{XN}}{\theta_{XL}}\sigma_X}{\lambda_{XL}}\right) < 0, \text{ where } \overset{*}{p}_N = \frac{t}{p_N} > 0 \qquad (32)$$

$$\overset{*}{Y} = \overset{*}{p}_N\left(-\eta^S_{Y,p_Y}\frac{\theta_{XN}}{\theta_{XL}}\right) > 0 \qquad (33)$$

$$\overset{*}{N} = \overset{*}{p}_N\left(-\frac{\sigma_X}{\theta_{XL}} + \frac{\lambda_{YL}}{\lambda_{XL}}\frac{\theta_{XN}}{\theta_{XL}}\eta^S_{Y,p_Y}\right) < 0 \qquad (34)$$

Since $\overset{*}{p}_Y < 0$, consumption of the non-traded, non-polluting good rises in the city. Production of X and use of N decline with the increase in factor cost. Note the two parameters involved are η^S_{Y,p_Y} and σ_X. The first indicates only substitution effects in consumption are present. We have taxed N which, given a gixed p_X, means p_L must <u>decline</u> which in turn means p_Y must fall in consumption. But the tax proceeds or the income effect of the price and income changes are paid back to labourers leaving only pure substitution effects in consumption. The σ_X indicates the degree of substitution in production of N and L; the higher σ_X, the more the use of N declines in production.

To evaluate the changes in welfare of city inhabitants, we must know consumption changes of X,Z,Y, and P. $\overset{*}{Y}$ is given in (33). From equation (12) we know $\overset{*}{P} = \varepsilon_N \overset{*}{N}$ where $\varepsilon_N = \frac{dP}{dN} \cdot \frac{N}{P}$ and therefore from (34)

$$\overset{*}{P} = \overset{*}{p}_N \varepsilon_N \left(- \frac{\sigma_X}{\theta_{XL}} + \frac{\lambda_{YL}}{\lambda_{XL}} \frac{\theta_{XN}}{\theta_{XL}} \eta^S_{Y,p_Y}\right) < 0 \qquad (35)$$

The greater σ_X and hence the more N is reduced with the tax, the more P will fall.

Putting equation (14) for the balance of payments in rate of change form we get $p_X X\overset{*}{X} - p_X X^C \overset{*}{X}{}^C = p_Z Z\overset{*}{Z} + p_N N\overset{*}{N}$. Substituting in (32) for $\overset{*}{X}$, (34) for $\overset{*}{N}$, and (28) for $\overset{*}{Z}$, we get

$$\overset{*}{X}{}^C = \overset{*}{p}_N\left(- \frac{\theta_{XN}}{\theta_{XL}} \eta^S_{X,p_Y}\right) < 0 \quad \text{if} \quad \eta^S_{X,p_Y} > 0 \qquad (36)^{17}$$

We can then solve for

$$\overset{*}{Z} = \overset{*}{p}_N\left(- \frac{\theta_{XN}}{\theta_{XL}} \eta^S_{Z,p_Y}\right) < 0 \quad \text{if} \quad \eta^S_{Z,p_Y} > 0 \qquad (37)$$

Providing all goods are substitutes in consumption, we know X^C and Z have declined in consumption whereas as Y and -P have risen. In net, we are going to hypothesize that utility levels will probably rise.

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17. $\overset{*}{X}{}^C$ is actually solved for as $\overset{*}{X}{}^C = \overset{*}{p}_N[\frac{\theta_{XN}}{\theta_{XL}} \; 1/k_X)\cdot(k_Y \eta^S_{Y,p_Y} + k_Z \eta^S_Y, p_Z)]$ but consumer theory tells us $\sum_i k_i \eta^S_{ji} = 0$.

From equation (13) for per person utility we know[18]

$$\Delta U \approx \frac{1}{L}(\partial U/\partial X\ \Delta X^C + \partial U/\partial Y\ \Delta Y + \partial U/\partial Z\ \Delta Z^C) + \partial U/\partial P \Delta P$$

From first order conditions of utility maximization we know for example $\partial U/\partial X = \lambda p_X$ where λ is the marginal utility of income. Therefore

$$\frac{\Delta U}{\lambda} \approx \frac{1}{L}[p_X X^C(\frac{\Delta X^C}{X^C}) + p_Y Y(\frac{\Delta Y}{Y}) + p_Z Z^C(\frac{\Delta Z^C}{Z^C})] + (\partial U/\partial P\ 1/\lambda)P(\frac{\Delta P}{P})$$

Substituting in values for the rates of change from (33), (35), (36) and (37) yields

$$\frac{\Delta U}{\lambda} \approx \overset{*}{p}_N[-p_L\frac{\theta_{XN}}{\theta_{XL}}(k_Y\eta^S_{Y,p_Y} + k_Z\eta^S_{Z,p_Y} + k_X\eta^S_{X,p_Y}) + P(\frac{\partial U}{\partial P}\cdot\frac{1}{\lambda})\epsilon_N\ \cdot$$
$$(-\frac{\sigma_X}{\theta_{XL}} + \frac{\lambda_{YL}}{\lambda_{XL}}\frac{\theta_{XN}}{\theta_{XL}}\eta^S_{Y,p_Y})]$$

But $\sum_i k_i\eta^S_{jpi} = 0$. Therefore

$$\frac{\Delta U}{\lambda} \approx \overset{*}{P}_N\ P(\frac{\partial U}{\partial P}\cdot\frac{1}{\lambda})\epsilon_N(-\frac{\sigma_X}{\theta_{XL}} + \frac{\lambda_{YL}}{\lambda_{XL}}\frac{\theta_{XN}}{\theta_{XL}}\eta^S_{Y,p_Y}) > 0 \qquad (38)$$

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18. Following Harberger (1970), if we take the second term of the Taylor expansion as well, in generalized notation

$$\Delta U = \sum_i U_i\Delta M_i + \tfrac{1}{2}\sum_i \Delta U_i\Delta M_i$$

where U_i is the marginal utility of i and M_i a good. Differentiating first order conditions yields $\Delta U_i = \lambda\Delta p_i + p_i\Delta\lambda + \Delta p_i\Delta\lambda$. Upon substitution, neglecting third order terms we have $\Delta U/(\lambda + \frac{1}{2}\Delta\lambda) \approx \sum_i p_i\Delta M_i + \frac{1}{2}\sum_i \Delta p_i\Delta M_i$.

In our example this reduces to $\Delta U/(\lambda+\frac{1}{2}\Delta\lambda) \approx P(\frac{\partial U}{\partial P} + \frac{1}{2}(\Delta\frac{\partial U}{\partial P}))$.

$$(\lambda + \tfrac{1}{2}\Delta\lambda)\epsilon_N(-\frac{\sigma_X}{\theta_{XL}} + \frac{\lambda_{YL}}{\lambda_{XL}}\frac{\theta_{XN}}{\theta_{XL}}\eta^S_{Y,p_Y})\overset{*}{p}_N + \tfrac{1}{2}p_Y Y(\overset{*}{p}_N)^2(\frac{\theta_{XN}}{\theta_{XL}})^2\eta^S_{Y,p_Y}$$

Now it is no longer absolutely certain that the change in utility is positive. Presumably the latter term could outweigh the former. We hypothesize otherwise.

If this approximation of the money value of change in utility is correct in directional sign (see footnote on page 24), then utility in the city increases.

Intuitively this makes sense. The basic city resource is labour, which produces goods for its own consumption and to trade for other goods. Given only basic resources of labour, a Pareto-optimal tax should by definition improve welfare. We tax the use of an input the city imports. The effect of the tax is to reorder production priorities in the city away from X to Y. Since the tax proceeds are returned to labour, only substitution effects in consumption are present and we showed by approximation that they net out in terms of affecting utility. The lose from utilizing less imported N is of second order magnitude. The main net result is a Pareto-optimal reduction in the imported input and the accompanying positive benefits from the optimal reduction in pollution.

If we were to consider situations where $f_i(L_i,N_i)$ and $F(L_Y)$ were not homogeneous of degree one, our result would be weakened. If the production of X were to have industry increasing returns to scale, apart from pollution, and Y to have decreasing returns to scale as we originally hypothesized, then the reduction of X and increase in Y would have adverse effects on labour productivity and the production potential of the city.

Given utility levels in the city rise, we can extend the analysis to the total economy. If utility levels rise, city size will increase as labour immigrates to enjoy higher utility levels in that city. City output of Y will further increase and X output will increase relative to its post-tax level. If pollution is optimally taxed in all cities producing X, but remains below pre-tax levels in all cities even when they increase in size, then we are assured that economy production of X will fall. Note that since X is being taxed throughout the economy by the standard analysis, N usage and X output for the economy will fall, as resources are shifted to less costly production.

It is also possible that ultimately X production in the city will rise, given the city is a more efficient production-consumption unit and immigration may be large. When N is optimally taxed in all cities, for X output to fall in the economy but city production of X to rise, there must be a decline in the number of cities producing X. Given initial multiple cities, the <u>optimum city size</u> of that type of city has <u>risen</u> and hence the number of that type of city may decline. Even if there is only one initial city producing X and economy X production falls there is no reason for the city population to fall. Production priorities have been reordered away from X to Y in the city and this reordering towards city non-traded goods should be maintained. X labour usage will fall but Y and total city labour usage will rise.

Note that, if any of the situations in part I prevail, our arguments are strengthened. There, the <u>first</u> round effect may be that city and economy output of X as well as Y will rise, since the initial equilibrium is inefficient as well as non-optimal. Given the arguments in this section, the second round effect will be to increase city size further.

<u>Conclusion</u>

In this paper we examined optimum city size in the presence of external diseconomies. The situations we examined were fairly specific but the conceptual conclusions and suggestions for future work are quite general:

a. When examining external diseconomies, production conditions must be carefully reviewed before one can conclude that Pareto-optimum production of the source of the externality is less than its competitive production. We saw that pollution could be so pervasive that the social marginal product of the input causing pollution would be negative under competition. A reduction in the input might then lead to increased output in the Pareto-optimum solution. Secondly by rearranging output between firms that pollute differently or are affected differently by pollution total output could rise following the imposition of the optimal tax.

Immediate parallels for other types of externalities spring to mind, in particular for congestion. Is the effect of optimally pricing congestion, to lower total traffic volume and hence (?) city size? Johnson (1964) has shown when traffic density becomes very high, traffic flow or volume may actually decrease as density and congestion rise (this occurs at about eleven miles per hour). Then an optimal congestion tax while lowering traffic density would increase traffic volume at a reduced price. Furthermore by taxing vehicles differentially, traffic composition might be changed at peak hours, increasing automobiles relative to trucks on intra-city expressways. Recently Mohring (1970) has shown that a full analysis would compare the optimal pricing-investment scheme with the existing solution of gasoline taxes. Switching to a time variable congestion price would most likely reduce off-peak prices while increasing peak prices relative to the competitive price including gasoline taxes. A complete analysis would examine how average price changes for all users, how optimal capacity of roads might change, the effect of a different financing scheme, etc. Whether optimal total traffic flows and average prices rise or decline is uncertain.[19]

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19. Drawing from Mohring's (1970) analysis, suppose all expansion is initially gasoline financed and investment occurs so that the marginal

b. In discussing the externality-city size question it must be realized we are discussing city size not firm/industry output. The city produces a variety of goods and is a consumption as well as production center. The tax on pollution ultimately affects consumers and as such will increase utility levels. Productive capacity (in the export good industry) may be reduced but so is the externality and the negative amenity income of city inhabitants. Optimal city size will increase.

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19. (continued)

benefits (reduction in total travel costs) of investment equal the marginal costs of investment. (This is not the second-best solution; from Mohring, pp. 700 and 701, marginal benefits given gasoline taxes should exceed marginal costs.) Replacing the gasoline tax with a congestion tax will raise the peak prices and lower the off-peak prices. Increasing the peak price will reduce peak usage and hence the marginal benefits of investment. Optimal capacity may fall. A complete answer would consider this, the effect upon total traffic volume of the peak increase in price but the off-peak decrease and the substitutes (rail and bus express) for peak vs non-peak users. Presumably the latter which are recreationally oriented have fewer, if any, substitutes.

Footnotes

1. See Sandquist (1970) or Mills and de Ferranti (1970). Mills and de Ferranti also question the standard conclusion.

2. See Kneese (1971) for a description of the multiple forms and effects of pollution.

3. See Baumol and Oates (1971) and Bohm (1971) for a description of pricing schemes and other methods of pollution control.

4. Henderson does not state the market achieves this efficient city size. His analysis and any reasons for non-optimality are independent of the analysis in this paper or external diseconomies.

5. This result can be derived in a Loschian (1954) world where efficient city size is limited by the increasing transport costs of supplying larger and larger rural areas as a city size increases.

6. Henderson's (1972) reasoning is as follows. If the production of two export goods or bundles of export goods involve no benefits from a common location such as utilizing a common specialized labour force or locationally fixed intermediate input, then it may be beneficial to locate the production of these goods in different cities. Locating their production together increases the labour force that must be housed and that adds to average commuting costs in the city while not increasing efficiency in the production of the traded goods. Separating the production of the two goods into different cities lowers the increase in per person commuting costs <u>relative</u> to scale economy exploitation <u>per unit of traded good</u> output, as traded good production and city size rise. Separating the two industries of course involves further transportation expenditures due to inter-city trade and a complete analysis would have to account for these costs.

7. If the city-specialization argument is not accepted and it is viewed that cities, under our simple assumptions, would produce all goods, our argument is even stronger! Optimally pricing the good causing externalities will only change the production ratios of goods in the economy and city. This says nothing, <u>a priori</u>, about the effect on city size.

8. It is possible with an unpriced factor which is <u>non-rivalrous</u> that the rent will be zero.

9. Meade (1952) "assigns" the rent of the unpriced factor to capital which, if the unpriced factor is an external diseconomy, is a negative rent. Given perfect capital markets, the negative rent means capital is underutilized, since its social marginal product is less than the market price which is equal to the private marginal product plus the negative rent. To equate private and social marginal products, Meade suggests subsidizing capital, from the tax proceeds received from taxing the source of the externality. Mishan (1965) has criticized both the automatic assignment of rent to capital versus other factors, as well as subsidizing capital rather than firm output.

10. See Meade (1952) in his atmosphere case or Chipman (1970).

11. Tolley (1969) has a similar example but does not make the negative social marginal product condition explicit. To quote Tolley, "A manifestation of the gain from reducing the externality is likely to be that the production function shift....lowering costs, is greater than the effect of the tax as a cost-raising item."

12. By inefficient, we mean if resources were diverted from national production of N, not only could production of X and Y or, at least, an index of total output of the two goods rise, but the diverted resources could be used in production of other additional consumer goods. For the same initial resources by rearranging production conditions, total output of all goods could rise.

13. This argument utilizes Schall's (1972) work.

14. See, for example, Henderson and Quandt (1971), p. 70.

15. Allen's (1968) condition occurs when, for $M = \frac{p_N N}{p_X X}$, σ = the elasticity of substitution, and η = elasticity of demand,

$$(\partial L/\partial p_N)\cdot(p_N/L) \;=\; m(\sigma-\eta) > 0, \text{ if } \sigma > \eta.$$

16. If the problem is formulated as an unpriced input, the analysis is even more complicated. For example, if there is a negative rent from the unpriced pollution, and if it is "assigned" to labour in the X and Y industry leading to too high a social marginal product of labour and too low labour usage, following Meade (1952), we would subsidize labour usage (or firm output) from the tax proceeds from N taxation. Then the factor substitution effect is strongly in favour of raising total labour usage in X production!

17. $\overset{*}{X}{}^C$ is actually solved for as $\overset{*}{X}{}^C = \overset{*}{p}_N[\frac{\theta_{XN}}{\theta_{XL}} 1/k_X)\cdot(k_Y \overset{S}{Y}, p_Y + k_Z\eta^S_Y, p_Z)]$ but consumer theory tells us $\sum_i k_i\eta^S_{ji} = 0$.

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In our example this reduced to $\Delta U/(\lambda+\tfrac{1}{2}\Delta\lambda) \approx P(\frac{\partial U}{\partial P} + \tfrac{1}{2}(\Delta\frac{\partial U}{\partial P}))$.

$$(\lambda + \tfrac{1}{2}\Delta\lambda)\;{}_N(-\frac{\sigma_X}{\theta_{XL}} + \frac{\lambda_{YL}}{\lambda_{XL}}\frac{\theta_{XN}}{\theta_{XL}}\eta^S_{Y,p_Y})\overset{*}{p}_N + \tfrac{1}{2}p_Y Y(\overset{*}{p}_N)^2(\frac{\theta_{XN}}{\theta_{XL}})^2\eta^S_{Y,p_Y}$$

18. Continued.

Now it is no longer absolutely certain that the change in utility is positive. Presumably the latter term could outweigh the former. We hypothesize otherwise.

19. Drawing from Mohring's (1970) analysis, suppose all expansion is initially gasoline financed and investment occurs so that the marginal benefits (reduction in total travel costs) of investment equal the marginal costs of investment. (This is not the second-best solution; from Mohring, pp. 700 and 701, marginal benefits given gasoline taxes should exceed marginal costs.) Replacing the gasoline tax with a congestion tax will raise the peak prices and lower the off-peak prices. Increasing the peak price will reduce peak usage and hence the marginal benefits of investment. Optimal capacity may fall. A complete answer would consider this, the effect upon total traffic volume of the peak increase in price but the off-peak decrease and the substitutes (rail and bus express) for peak vs non-peak users. Presumably the latter which are recreationally oriented have fewer, if any, substitutes.